Preprocessing in the PS space for on-shore seismic processing: removing ground roll and ghosts without damaging the reflection data

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SUMMARY

Prerequisites (e.g., the removal of the reference wave and the ghosts) are important for on-shore seismic processing. This paper derives an elastic Green's theorem wave separation algorithm for data in the PS space. Applying the algorithm presented in this paper, both the reference waves (including the direct wave and the surface wave) and the ghosts can be effectively removed. The method is tested on a layered elastic earth model. The results indicate its effectiveness for reducing the ground roll and ghosts, and without harming the up-going reflection, in preparation for on-shore processing.

INTRODUCTION

On-shore seismic exploration and processing seeks to use reflection data (the scattered wavefield) to detect the subsurface information. The measured total wavefield consists of the reflection data and the reference wavefield that contains the surface wave/ground roll; hence, it is necessary to separate the reference wave and the scattered wave. Filtering methods are typically employed to remove the reference wave, particularly the ground roll, but at the expense of damaging reflection data when ground roll is interfering with the scattered wavefield. As a flexible and useful tool, Green's theorem provides methods that can separate the reference wave from the reflection data without damaging the reflection. The application of these methods represents the unique advantages for off-shore plays (e.g., Weglein et al., 2002; Zhang, 2007; Mayhan et al., 2011; Mayhan and Weglein, 2013; Tang et al., 2013; Yang et al., 2013).

For on-shore plays, one of the key problems is the complex and laterally varying near surface. Our study starts with a simpler example, by assuming the space just below the free surface is homogeneous and known, but the earth below the measurement surface is unknown and heterogenous. Wu and Weglein (2014) derive the elastic Green's theorem reference and scattered wave separation algorithm for data in the PS space, and successfully test the algorithm on an initial model without subsurface reflectors. In this paper, for more realistic situation, we add one reflector in the tested model so that the measured data contain both the reference wave and the scattered wave.

In addition, for buried sources and receivers, not only up-going waves are in the reflection data but also ghosts, whose existence can cause notches in the spectrum. Thus, after obtaining the reflection data, removing the ghosts from the reflection data is another prerequisite. In this study, we will assume the source is located slightly above the air/earth surface (could be infinitely close, or on the air/earth surface), and the receivers are slightly beneath the air/earth surface. Therefore, there are receiver ghosts but no source ghosts in our study. Green's theorem can also be applied for deghosting, by taking a whole space homogeneous elastic medium as reference. A numerical test is shown to examine the accuracy of the deghosting algorithm.

GREEN'S THEOREM WAVE SEPARATION THEORY IN THE PS SPACE

Background of 2D elastic wave theory

We are deriving the wave separation method for on-shore application and starting with the elastic formulation. For convenience, the basis is changed from $\mathbf{u} = \begin{pmatrix} u_x \\ u_z \end{pmatrix}$ to $\Phi = \begin{pmatrix} \phi^P \\ \phi^S \end{pmatrix}$. $\mathbf{u}$ represents the displacement, consisting of x and z components; whereas Φ has P-wave and S-wave components.

In the PS space, the basic wave equations (Weglein and Stolt, 1995; Zhang, 2006) are

$$\begin{aligned} \hat{\mathbf{L}}\Phi &= \mathbf{F} \\ \hat{\mathbf{L}}\hat{\mathbf{G}} &= \delta, \\ \hat{\mathbf{L}}_0\Phi_0 &= \mathbf{F}, \\ \hat{\mathbf{L}}_0\hat{\mathbf{G}}_0 &= \delta, \end{aligned} \tag{1}$$

where $\hat{\mathbf{L}}$ and $\hat{\mathbf{L}}_0$ are the differential operators describing the properties of the actual medium and the reference medium, respectively. $\mathbf{F}$ is the source term. $\hat{\mathbf{G}}$ and $\hat{\mathbf{G}}_0$ are the Green's function operators for the actual and reference media, respectively.

The basic forms of these equations are the same as those for the acoustic case. On the basis of the successful applications of Green's theorem wave separation to the acoustic case (e.g., Zhang, 2007; Mayhan et al., 2011), it is feasible to apply the Green's theorem wave separation algorithm to the elastic world in a similar way.

Description of the model for wave separation

As seen in Figure 1, the model here consists of a half space of air and a half space of elastic earth. Receivers are buried in the earth, and the source is located slightly above the free surface (F.S.). The measurement surface (M.S.) can be infinitely close to the free surface, like the on-surface acquisition, or several meters below the free surface, like the buried-receiver acquisition; however, the receivers are coupled with the elastic medium in both situations.

Reference and scattered wave separation

The reference wave is the wave in the reference medium. It's useful for the purpose of exploration seismology to choose the reference medium to agree with the actual earth at and above the measurement surface. If we assume the actual earth

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has known and homogeneous near surface properties, the simplest reference medium can be chosen as discontinuous two half-spaces, homogeneous air over homogeneous elastic earth (see Figure 2). There are two sources acting on the reference medium (see Figure 3). One is the active source (S_1), generating the reference wave; the other is the earth heterogeneity, or passive source (S_2), generating the scattered wave.

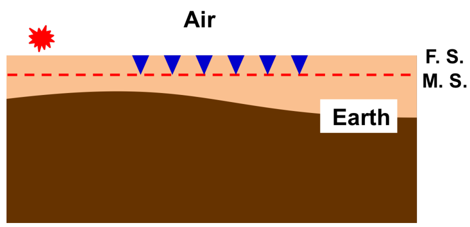

Figure 1: A generic model describing the land experiment

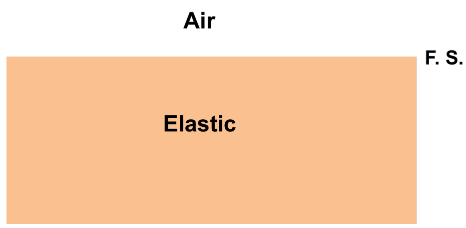

Figure 2: Reference medium for reference and scattered wave separation

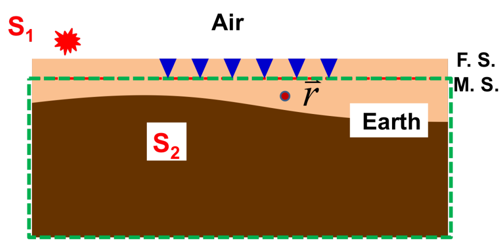

Figure 3: Reference wave separated from the total wavefield with surface integral along the dash line, for **r** below the M.S.

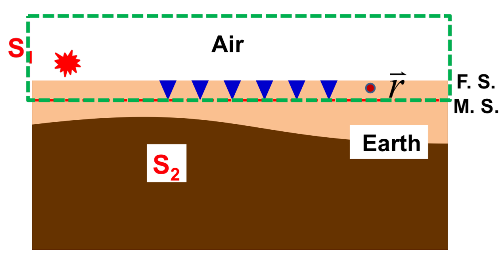

Figure 4: Scattered wave separated from the total wavefield with surface integral along the dash line, for **r** above the M.S.

Wu and Weglein (2014) have shown that, by applying Green's theorem, a semi-infinite surface integral upper bounded by the measurement surface will separate the reference wavefield Φ_0 from the total wavefield Φ, for the evaluation point **r** inside the volume and below the measurement surface (see Figure 3); whereas, the surface integral lower bounded by the measurement surface will separate the scattered wave Φ_s from Φ, for **r** above the measurement surface (see Figure 4).

The Green's theorem based formula for reference and scattered wave separation in the space-frequency (x,ω) domain is

$$\oint \left(\Phi(\mathbf{r}',\mathbf{r}_s,\omega)\cdot\nabla'\hat{\mathbf{G}}_0(\mathbf{r}',\mathbf{r},\omega) - \nabla'\Phi(\mathbf{r}',\mathbf{r}_s,\omega)\cdot\hat{\mathbf{G}}_0(\mathbf{r}',\mathbf{r},\omega)\right)\cdot\hat{n}dS'$$
$$= \begin{cases} \Phi_0(\mathbf{r},\mathbf{r}_s,\omega) & \mathbf{r} \text{ is below the M.S.,} \\ \Phi_s(\mathbf{r},\mathbf{r}_s,\omega) & \mathbf{r} \text{ is above the M.S.,} \end{cases} \tag{2}$$

where the symbol $'\cdot'$ represents a tensor product.

$\Phi_0(\mathbf{r},\mathbf{r}_s) = \begin{pmatrix} \Phi_0^P(\mathbf{r},\mathbf{r}_s) \\ \Phi_0^S(\mathbf{r},\mathbf{r}_s) \end{pmatrix}$, $\Phi_s(\mathbf{r},\mathbf{r}_s) = \begin{pmatrix} \Phi_s^P(\mathbf{r},\mathbf{r}_s) \\ \Phi_s^S(\mathbf{r},\mathbf{r}_s) \end{pmatrix}$, $\Phi(\mathbf{r},\mathbf{r}_s) = \begin{pmatrix} \Phi^P(\mathbf{r},\mathbf{r}_s) \\ \Phi^S(\mathbf{r},\mathbf{r}_s) \end{pmatrix}$, and Green's function $\hat{\mathbf{G}}_0(\mathbf{r}',\mathbf{r},\omega)$ for the reference medium is

$$\hat{\mathbf{G}}_0(\mathbf{r}',\mathbf{r},\omega)$$
$$= \begin{pmatrix} \hat{G}_0^P(\mathbf{r}',\mathbf{r},\omega)+\hat{G}_0^{PP}(\mathbf{r}',\mathbf{r},\omega) & \hat{G}_0^{PS}(\mathbf{r}',\mathbf{r},\omega) \\ \hat{G}_0^{SP}(\mathbf{r}',\mathbf{r},\omega) & \hat{G}_0^S(\mathbf{r}',\mathbf{r},\omega)+\hat{G}_0^{SS}(\mathbf{r}',\mathbf{r},\omega) \end{pmatrix}$$
$$= \frac{1}{2\pi}\int e^{ik_x(x'-x)}dk_x \begin{pmatrix} \frac{e^{i\nu_2|z'-z|}}{2i\nu_2} + \acute{P}\grave{P}\frac{e^{i\nu_2 z}e^{i\nu_2 z'}}{2i\nu_2} & \acute{S}\grave{P}\frac{e^{i\eta_2 z}e^{i\nu_2 z'}}{2i\eta_2} \\ \acute{P}\grave{S}\frac{e^{i\nu_2 z}e^{i\eta_2 z'}}{2i\nu_2} & \frac{e^{i\eta_2|z'-z|}}{2i\eta_2} + \acute{S}\grave{S}\frac{e^{i\eta_2 z}e^{i\eta_2 z'}}{2i\eta_2} \end{pmatrix}, \tag{3}$$

where $\acute{P}\grave{P}$, $\acute{P}\grave{S}$, $\acute{S}\grave{P}$, $\acute{S}\grave{S}$ represent the reflection coefficients along the air-elastic boundary, the subscript '2' represents the elastic half-space, and

$$\nu_2 = \begin{cases} \sqrt{k_{\alpha_2}^2 - k_x^2} & \text{if } k_x < k_{\alpha_2} \\ i\sqrt{k_x^2 - k_{\alpha_2}^2} & \text{if } k_x > k_{\alpha_2} \end{cases} \qquad k_{\alpha_2} = \frac{\omega}{\alpha_2},$$

$$\eta_2 = \begin{cases} \sqrt{k_{\beta_2}^2 - k_x^2} & \text{if } k_x < k_{\beta_2} \\ i\sqrt{k_x^2 - k_{\beta_2}^2} & \text{if } k_x > k_{\beta_2} \end{cases} \qquad k_{\beta_2} = \frac{\omega}{\beta_2}.$$

α_2 and β_2 represent P and S velocities in the elastic medium.

If the measurement surface is horizontal, the outward normal vector $\hat{n} = (0, \mp 1)$, where -1 is corresponding to the situation as shown in Figure 3, and 1 is for Figure 4.

Using reciprocity of the Green's function and Fourier transforming over x in Equation 2 with $\int e^{-ik_x x}dx$, the formula in the wavenumber-frequency (k_x,ω) domain will be

$$\left[\tilde{\Phi}(k_x,z,\mathbf{r}_s)\cdot\partial_{z'}\tilde{\hat{\mathbf{G}}}_0^T(k_x,z,z') - \partial_{z'}\tilde{\Phi}(k_x,z,\mathbf{r}_s)\cdot\tilde{\hat{\mathbf{G}}}_0^T(k_x,z,z')\right]\Big|_{z'=\varepsilon_g}$$
$$= \begin{cases} -\tilde{\Phi}_0(k_x,z,\mathbf{r}_s) & z \geq \varepsilon_g^+, \\ \tilde{\Phi}_s(k_x,z,\mathbf{r}_s) & z \leq \varepsilon_g^-. \end{cases} \tag{4}$$

Tildes represent the terms in k_x domain, $\tilde{\hat{\mathbf{G}}}_0^T$ is the transpose of $\tilde{\hat{\mathbf{G}}}_0$, and ε_g is the receiver's depth. z' is evaluated at ε_g.

It is deserving emphasis that applying the algorithm in the (k_x,ω) domain, we can arrange to locate the output point **r** on the measurement surface to separate the actual measured data into the reference wave and the scattered wave. We can obtain the reference wave of measured data by choosing **r** on the measurement surface to be part of the volume below; otherwise, we can obtain the scattered wave of measurement by

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choosing $\mathbf{r}$ on the measurement surface to be part of the volume above.

Deghosting the reflection data

Green's theorem can be further applied for deghosting the reflection data. For this time, the property along the measurement surface is assumed to be homogeneous and known. The reference medium is a whole space of homogeneous elastic (see Figure 5), whose properties are consistent with the actual earth along the measurement surface. Similar to the theory of reference and scattered wave separation, a semi-infinite surface integral lower bounded by the measurement surface will separate the up-going wave Φ_{up} from the scattered wave Φ_s, for $\mathbf{r}$ inside the volume (see Figure 6).

The elastic Green's theorem deghosting formula in space-frequency (x,ω) domain is

$$\Phi_{up}(\mathbf{r},\mathbf{r}_s,\omega) = \oint \left(\Phi_s(\mathbf{r}',\mathbf{r}_s,\omega)\cdot\nabla'\hat{\mathbf{G}}_0(\mathbf{r}',\mathbf{r},\omega) - \nabla'\Phi_s(\mathbf{r}',\mathbf{r}_s,\omega)\cdot\hat{\mathbf{G}}_0(\mathbf{r}',\mathbf{r},\omega)\right)\cdot\hat{n}dS', \tag{5}$$

where $\Phi_{up}(\mathbf{r},\mathbf{r}_s) = \begin{pmatrix}\Phi^P_{up}(\mathbf{r},\mathbf{r}_s)\\ \Phi^S_{up}(\mathbf{r},\mathbf{r}_s)\end{pmatrix}$ is the separated up wave, and $\hat{\mathbf{G}}_0(\mathbf{r}',\mathbf{r},\omega)$ of the reference medium is

$$\begin{aligned}\hat{\mathbf{G}}_0(\mathbf{r}',\mathbf{r},\omega) &= \begin{pmatrix}\hat{G}^P_0(\mathbf{r}',\mathbf{r},\omega) & 0\\ 0 & \hat{G}^S_0(\mathbf{r}',\mathbf{r},\omega)\end{pmatrix}\\ &= \frac{1}{2\pi}\int e^{ik_x(x'-x)}dk_x \begin{pmatrix}\frac{e^{i\nu_2|z'-z|}}{2i\nu_2} & 0\\ 0 & \frac{e^{i\eta_2|z'-z|}}{2i\eta_2}\end{pmatrix}.\end{aligned} \tag{6}$$

Similarly, we can Fourier transform Equation 5 to (k_x,ω) domain if the measurement surface is horizontal and flat.

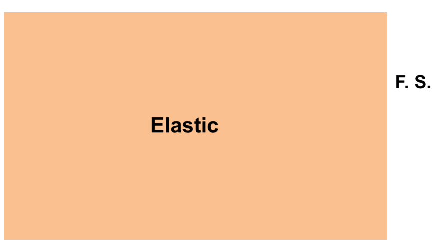

Figure 5: Reference medium for deghosting

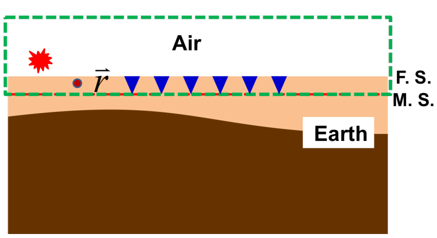

Figure 6: Up wave separated from the reflection data with surface integral along the dash line, for $\mathbf{r}$ above the M.S.

NUMERICAL EVALUATION

The reference and scattered wave separation and deghosting methods that are developed in the paper are tested on an air/elastic-earth model. As shown in Figure 7, the model consists of a half-space of air and a half-space of two layered elastic earth, and the parameters are listed in Table 1. A P source is applied on the free surface. The receivers are 5m below the free surface and they record both P and S waves. The output point $\mathbf{r}$ is on the measurement surface and the formula in (k_x,ω) domain is applied.

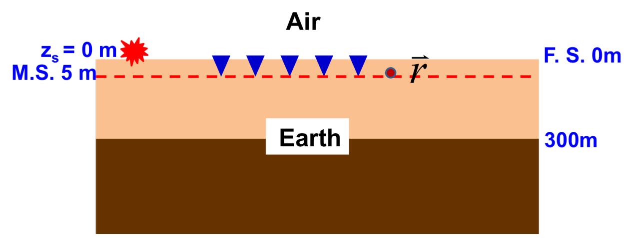

Figure 7: The air/elastic model for the numerical tests

Layer Number	P Velocity (m/s)	S Velocity (m/s)	Density (kg/m^3)
1	340	0	3
2	2000	1200	1500
3	4000	3000	1800

Table 1: The air/elastic model parameters

Reference and scattered wave separation

The data of total wavefields in the PS space (Figure 8(a) for Φ^P and Figure 8(d) for Φ^S) are created with the analytic forms, consisting of the reflection data and the reference wave that includes the ground roll. They will be substituted into Equation 4 for reference and scattered wave separation. Since the output $\mathbf{r}$ is on the measurement surface, we can obtain the reference wave (Figure 8(b) for Φ^P_0 and Figure 8(e) for Φ^S_0) of measured data by choosing $\mathbf{r}$ to be part of volume below, and we can also obtain the scattered wave (Figure 8(c) for Φ^P_s and Figure 8(f) for Φ^S_s) of measured data by choosing $\mathbf{r}$ to be part of volume above. All the figures are in the same scales. Comparing the separated reference waves from Green's theorem wave separation algorithm with those of the input data, both their amplitudes and phases match well. The conclusion retains for the scattered waves.

Deghosting

The scattered wavefield (Figure 9(a) for Φ^P_s and Figure 9(d) for Φ^S_s) can be further separated into up-going and down-going waves. We apply the deghosting algorithm to separate the up wave (Figure 9(b) for separated Φ^P_{up} and Figure 9(e) for separated Φ^S_{up}), for the output point $\mathbf{r}$ on the measurement surface to be part of volume above. To evaluate the result, we analytically create the data with only up wave using the given model (Figure 9(c) for synthetic Φ^P_{up} and Figure 9(f) for synthetic Φ^S_{up}). Comparing Figure 9(b) with Figure 9(c) for P

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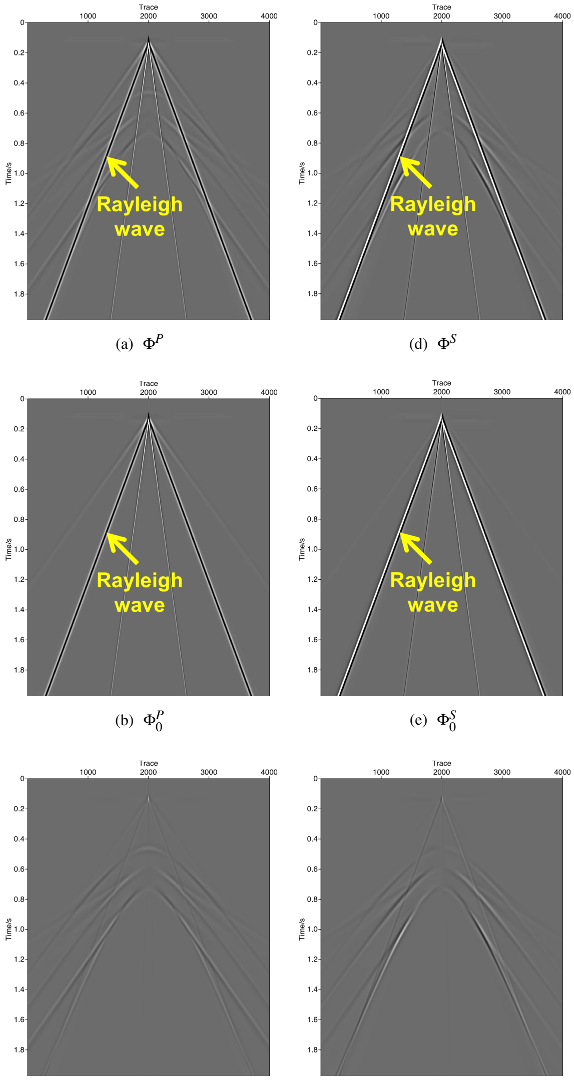


(a) Φ^P (d) Φ^S

(b) Φ^P_0 (e) Φ^S_0

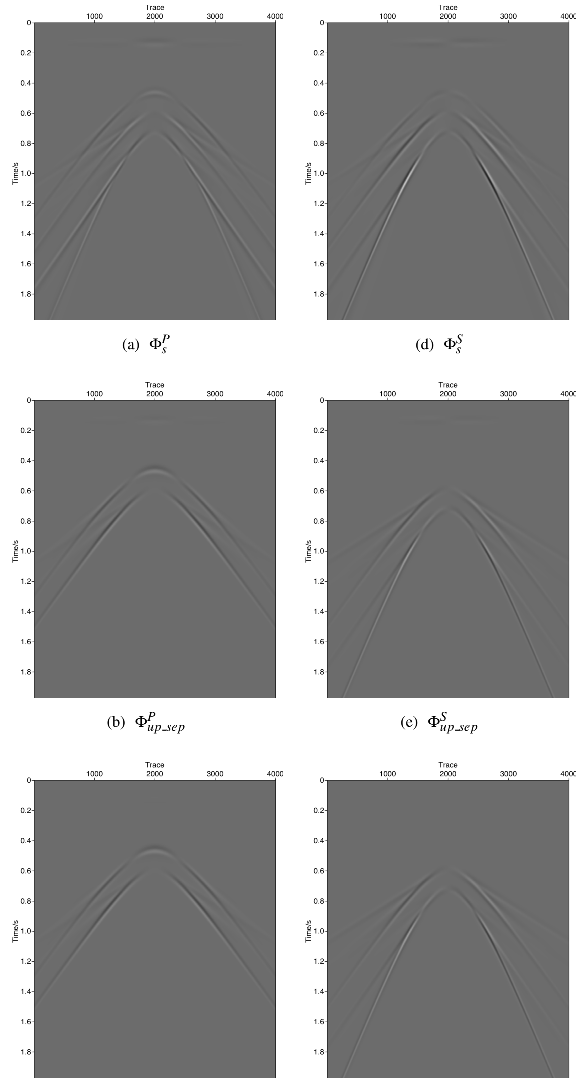
(c) Φ^P_s (f) Φ^S_s

Figure 8: Reference and scattered wave separation results. (a) is the total P wave; (b) is the separated P component of reference wave, by choosing **r** to be part of volume below; (c) is the separated P component of scattered wave, by choosing **r** to be part of volume above; (d) is the total S wave; (e) is the separated S component of reference wave; (f) is the separated S component of scattered wave.

components, and comparing Figure 9(e) with Figure 9(f) for S components, the up-going waves are effectively extracted.

CONCLUSION

We apply the elastic Green's theorem method to separate the reference and scattered wave and remove the ghosts of the reflection data. It has the potential to remove the ground roll, which is a major and serious issue and part of the reference wave, and to remove the ghosts, for on-shore experiments without damaging the reflection data. To make the method more readily applicable in practice, our research plan is in reducing the data requirements and pursuing an alternative approach without the need for near surface information.



(a) Φ^P_s (d) Φ^S_s

(b) $\Phi^P_{up\_sep}$ (e) $\Phi^S_{up\_sep}$

(c) $\Phi^P_{up\_syn}$ (f) $\Phi^S_{up\_syn}$

Figure 9: Deghosting results. (a) is P component of the input scattered wave; (b) is the separated P component of up-going wave, by choosing **r** to be part of volume above; (c) is the synthetic P component of up-going wave, with analytic form; (d) is S component of the input scattered wave; (e) is the separated S component of up-going wave; (f) is the synthetic S component of up-going wave.

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EDITED REFERENCES

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